Buy-Write or Put-Write,
An Active Portfolio to Strike it Right

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Abstract

As passive investment strategies, Buy-Write (Covered Call) and Collateralized Put-Write have complementary risk and return characteristics. When applied to broad stock market index such as the S&P 500 Index, they offer long term improved risk adjusted returns compared to the traditional Buy-and-Hold approach, although both Buy-Write and Put-Write strategies have periods of under-performance but in different trending markets. Historically, an intermediate or long term trend-following method, such as the Golden Cross/Black Cross Rule (50-day moving average cross above/below 200-day moving average), has been reasonably successful in avoiding bear market and participating in most of the rising regimes. Trying to combine favorable characteristics from covered /collateralized option writing strategies with a market timing scheme, this paper proposes an Active Leveraged Option-Overlay Portfolio (*ALOOP*) strategy, which switches between Buy-Write and Put-Write according to a robust trend-following rule.

Introducing a "Factor of Leverage" (f_l) to vary the portfolio's nominal market exposure of long under bullish expectation or short under bearish expectation, the active portfolio consists of: a portion of long position in the underlying stock index (S&P 500 Total Return Index is used as an example throughout this paper), written at-the-money (ATM) *Call* or *Put* options with coming month expiration(CBOE SPX Index Options are used as examples), and *Collateral Cash* invested in risk-free Treasury bills.

A two-state continuous model of the investable portfolio is presented with an active portfolio decomposition: a Buy-Write or Collateralized Put-Write component, and an f_l portion short or long position in the underlying equity index, and its funding long or short position in equal amount of Treasury bills. Based on *Black-Scholes* Option Pricing Theory

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and a *volatility skew* estimation, the model yields a first order approximation of the portfolio's Greek Letter *Delta* that calibrates well with existing Buy-Write and Put-Write indexes. As a measure of portfolio's equity exposure, *Delta* shows a deviation from the *Beta* of an active but long equity only (*LEO*) portfolio under the same trend-following rule. The analytical expression gives a mathematical explanation of the source of favorable returns for the active strategy: equity over-exposure in rising market due to *volatility skew*. However, a pitfall of under-performance can exist during precipitous market decline, such as those in the summer of 1998 and the fall of 2008. More frequent roll-over to written at-the-money (or out-of-money) call or put option is suggested as possible adjusting mechanism during periods of market stress.

The proposed *S&P 500 Total Return Index Golden Cross/Black Cross - Active Leveraged Option Overlay Portfolio* (*SPTR GCBC-ALOOP*) is back-tested based on combining three standard indexes: S&P 500 Total Return Index *(SPTR)*, CBOE SPX *Buy-Write* Index (*BXM*) and SPX *Put-Write* Index (*PUT*), for a period of 21.6 years: from 06/01/1988 (when CBOE's *BXM* and *PUT* indexes started with a value of 100) to 12/31/2009. At "Factor of Leverage" $f_l = 0.5$, the active portfolio (*SPTR GCBC-ALOOP)* achieves a compounded annualized return of *16.3%*, a Sharpe ratio of *0.95*, a Sortino ratio of *1.35,* all significantly better than those of the passive indexes: Buy-and-Hold *S&P 500 Total Return Index (SPTR)*, *PUT*, and *BXM*, and those of the nominal *Delta*-matched *S&P 500 Total Return Index Golden Cross - Long Equity Only* (*SPTR GC-LEO*) strategy. The maximum drawdown *22.88%* of *SPTR GCBC-ALOOP* is slightly higher than that of the *SPTR GC-LEO*, but much smaller than those of the passive indexes (see table below). Single dollar growth from 6/1/1988 reaches *26.22* dollars on 12/31/2009 for the proposed *SPTR GCBC-ALOOP* ($f_l = 0.5$), compared to *6.77* dollars for Buy-and-Hold *SPTR*, *7.66* dollars for *BXM*, *9.73* dollars for *PUT*, and *10.36* dollars for *SPTR GC-LEO* strategy.

	SPTR	BXM	PUT	SPTR GC-LEO	SPTR GCBC-ALOOP (f = 0.5)
Annualized Return	9.26%	9.89%	11.11%	12.36%	16.33%
Annualized Std Deviation	18.27%	12.74%	11.88%	11.43%	13.99%
Sharpe Ratio	0.3440	0.5425	0.6847	0.6841	0.9543
Sortino Ratio	0.4922	0.7509	0.9480	0.9899	1.3483
Beta	1.0000	0.6345	0.5693	0.4581	0.5563
USharpe Ratio	6.29%	15.57%	21.61%	25.82%	47.99%
Max Draw-Down	55.24%	40.14%	37.09%	19.10%	22.88%

During the back-tested 21.6 years period, the active portfolio would have required only *18* additional trades at *Golden Cross* or *Black Cross* days besides the monthly *SPX* options roll-over implied in *BXM* and *PUT* indexes. Under conservative transaction costs and market friction estimates, the active portfolio (*SPTR GCBC-ALOOP*) still out-performed the *SPX* passive strategies and *SPTR GC-LEO* on basis of both absolute return and risk-adjusted-returns.

Regarding the "Factor of Leverage" portfolio parameter, it is found that $f_l = 0.5$ yields a conveniently investable portfolio with nearly optimal risk-adjusted-return. The portfolio effectively allocates half of the assets to *Buy-and-Hold* the *S&P 500 Total Return Index* (*SPTR)* and another half in Treasury bills; rolls over written ATM *SPX* call or put options and rebalance portfolio on monthly option expiration Fridays; and swaps written call for put *SPX* options (or vice versa) and rebalance portfolio on *SPTR* Golden Cross (or Black Cross) trigger days.

Given the relative easiness to implement the proposed active option overlay portfolio approach for index investment, it has the potential for broad adoption. Thus it is of interest to examine the future sustainability of the out-performance observed in the 21.6 years of back-test with *S&P 500 Total Return Index* and *SPX Index Options*. Since the active strategy proposes a fixed portion (about half) of the portfolio to Buy-and-Hold the underlying equity index, it has a stabilization effect in the underlying asset market and potentially slows down the pace of a market decline. This provides a natural remedy for the discussed performance pitfall of the active strategy itself, and help to improve the order-nesses and efficiencies in both the underlying asset and option markets. On the other hand, amplified in option market relative to asset market, investor behavioral biases of fear or over-confidence could be the fundamental reason for the active strategy's hypothetical historical success. As long as these market structural characteristics and human nature in investing persist, the observed effectiveness of the current proposal could be expected to last, even if we have not found an active management's "holy grail", for decades to come.

List of Acronyms and Symbols

50 DMA	50-day Simple Moving Average
200 DMA	200-day Simple Moving Average
ALOOP	Active Leveraged Option Overlay Portfolio
GC	Golden Cross – occurs when 50DMA crosses above 200DMA
BC	Black Cross – occurs when 50DMA crosses below 200DMA
LEO	Long Equity Only portfolio
SPX	Standard and Poor 500 Index
SPTR	S&P 500 Total Return Index
BXM	CBOE Buy-Write Index
PUT	CBOE Put-Write Index
CPW	Collateralized Put-Write
CBW	Covered Buy-Write
C	Call Option
P	Put Option
B	Treasury bill
S	Underlying stock or stock index
f_l	Factor of Leverage for the portfolio
Π	Portfolio asset value
Δ	Portfolio Delta
δ	Delta of a Call Option
σ_{BS}	Implied Volatility according Black-Scholes Option Pricing Theory
$o(\cdot)$	Higher order terms in a Taylor series expansion
r_f	Annualized risk free rate, such as yield of 3-month Treasury Bill
q	Annualized dividend yield of a stock or stock index
T	Time to expiration for Call and Put options
F	Total market friction cost

Introduction & Motivation

A "Buy-Write" (*BW*) strategy, also known as a "covered call", is an investment strategy that an investor buys a stock or an option-able Index ETF (an index Exchange-Traded-Fund represents essentially a basket of stocks), and writes (or sells) call options to cover the underlying stock or index ETF position. The "Buy-Write" strategy has been widely used to enhance portfolio returns and to reduce volatility, typically in down markets when the option premium received compensates the price decline in the underlying equity position. The shortcoming of Buy-Write is that in strong rising markets, the upside gain of the underlying equity investment is reduced when the written call option expires in the money for an amount more than the option premium received. Compared to a passive "Buy-and-Hold" position in the underlying stock or index ETF, "Buy-Write" is likely to outperform in bear markets and under-perform in bull markets [1][2][3].

Complementarily, Collateralized Put-Write (*CPW*) is an investment strategy that systematically writes (or sell) put options, while holding Treasury bills in equal amount (matching the number of shares) of the notional value of the underlying stock or index ETF. Unlike put buyer's bearish stance to hold an insurance against downside risk, Put-Write has a long exposure to the underlying stock or index ETF. The profit for *CPW* strategy is thus capped at the sum of the put premium sold and the interest earned on the Treasury bills collateralizing the strategy. Put-write strategy is always profitable in rising market. Only in a scenario that the down-ward move of the underlying equity away from the strike price (during the put initiation and expiration days) is larger than the put premiums collected, Put-Write has a loss [4].

Standard market indexes [5] have been developed to track the performance of Buy-Write and Put-Write strategies. The CBOE S&P 500 Buy-Write Index (*BXM*) is a benchmark index based on a hypothetical Buy-Write strategy on the S&P 500 Index. The methodology of the *BXM* Index consists of buying an S&P 500 index portfolio, and writing next-month expiration, at-the-money S&P 500 "covered" call option on monthly option expiration Friday (the third Friday each month). The call option premium collected becomes part of the portfolio's total value. The written call is held until expiration - the third Friday of the following month when a new one-month, at-the-money call is written. An expired in-the-money option is settled in cash. Similarly, the CBOE S&P 500 Put-Write Index (*PUT*) systematically sells one-month at-the-money put options on the S&P 500 Index collateralized by a portfolio of Treasury bills. Both *BXM* and *PUT* start on 6/1/1988 with normalized value of 100. For the period of 21.6 years since then (till 12/31/2009), both *BXM* Index and *PUT* Index performed better than S&P 500 Total Return Index (*SPTR*), with higher annualized compounded return, Sharpe ratio and Sortino ratio, and lower total risk (return standard deviation), beta and maximum drawdown (see Table 3). *PUT* also notably out-performed *BXM* during the period because:

1. *SPTR* index has more up months than down months. Looking at monthly period between two consecutive option expiration Fridays, there are total 259 months in the period that the index data are available (from 6/1/1988 to 12/31/2009). Excluding months with less than 1% change (up or down) in *SPTR,* 134 months out of 205 were up, the rest 71 were down. Up-trending market is to the advantage of Put-Write that has a long bias, as such *PUT* index surpassed *BXM* in time weighted performance.

2. Slightly out of money puts and calls are used in *PUT* and *BXM* indexes when *SPX* can not exactly match a strike price available at option expiration Friday's close. As a result, *volatility skew* gives more premiums to *PUT* than *BXM* index.

Buy-Write and Collateralized Put-Write are the simplest form of an Option Overlay Portfolio (*OOP*). Since the underlying equity position in Buy-Write or the collateral Treasury bills position in Put-Write ensures a non-negative portfolio value, the written options in both are fully covered. Subject to options margin requirement, more general case of Leveraged Option Overlaid Portfolio (*LOOP*) exists when variable levels of underlying equity position and collateral Treasury Bills are used. The overlay using option contracts is essentially a risk arbitrage between the derivative market and underlying equity asset market. Positive returns and out-performance of a *LOOP* can be attributed to two mechanisms:

1. Statistically, implied volatilities in option market are larger than the realized volatilities of underlying stock or Index ETF, regardless of the direction of the underlying equity market, up or down. Even when the actual market move turns against the long or short bias (of Put-Write or Buy-Write) taken by *LOOP*, written

option position can be still profitable as long as the option premium collected at initiation can cover the in-money-ness of the written option contracts at expiration. Due to equity market *volatility skew* (that has been observed since 1987 market crash), this effect is more profound for Put-Write than for Buy-Write strategy.

2. At expiration, the written options in Buy-Write or Put-Write turn out-of-money or have reduced in-money-ness compared to their initiation. This happens when the type of the option overlay matches the direction of market move: a long-biased *LOOP* (such as Put-Write) in a rising market, or a short-biased *LOOP* (such as Buy-Write) in a down market. In case that ATM (at-the-money) option contracts were written at initiation, 100% of the option premiums collected are realized as gains.

As the first mechanism can compensate at least portion of the *ex post* loss of a mistaken long or short bias of *LOOP*, it is natural to introduce active market timing – a proposal of an Active Leveraged Option Overlay Portfolio (*ALOOP*) strategy: switch from a long biased *LOOP* to a short biased *LOOP* (e.g. from Buy-Write to Put-Write) or vice versa according to an active market timing rule. When active timing works well, the second mechanism ensures maximized profitability on written option positions. Otherwise in quiet or trend-less market, the first mechanism provides insurance for *ALOOP* to out-perform. Compared to the passive Buy-and-Hold strategy, however, *ALOOP* can still under-perform if the active timing turns out to be on the wrong side of the market and underlying equity drops or rises at option expiration beyond what written options' premium can cover. It is of interest to identify these situations in the back-test and design feasible operational remedies for active management practice.

Choice of Active Management Scheme for *LOOP*

Moving Average is one of the most versatile and widely used technical indicators for active management. Easily quantified and tested, it is the basis of most trending-following system today. The moving average is also a smoothing device with time lag. Once prices start to trend, using long range moving average becomes advantageous (than short term moving average) as they can avoid minor corrections or consolidations and ride with the major trend longer. However, it can have more delay in responding to trend reversal [6]. For

ALOOP, active signal generated from long term moving average can be less susceptible to signal delays in either the long side or the short side. This is because the shorting volatility mechanism of *ALOOP* (when implied volatilities of written options are larger than realized volatilities for the underlying equity index) can somewhat compensate opportunity loss due to signal delay. Besides avoiding excessive market friction or transaction costs, long term moving average based active scheme can also fit *ALOOP* better, as trading interval longer than monthly duration of option expiration can help *ALOOP* fully monetize negative option *Theta* [7] in a trend-less market.

As a popular "double crossover method", Golden Cross / Black Cross seems satisfy the requirement of an active management scheme for *ALOOP* on broad market index such as S&P 500 Index. Golden Cross refers to 50 Day Simple Moving Average (50 DMA) crossing above 200 Day Simple Moving Average (200DMA), while Black Cross refers to 50DMA crossing below 200DMA. Golden Cross/Black Cross (*GCBC*) technical scheme applied to daily close price of S&P 500 Total Return Index (*SPTR*) is implemented as the following (called *SPTR GC-LEO*, S&P 500 Total Return Index Golden Cross Long Equity Only strategy):

1. At the close of the day of Golden Cross (when 50DMA > 200DMA happen for the first day in *SPTR* price), a long position in *SPTR* is entered by liquidating all 3-month Treasury bills;

2. Long position in *SPTR* is held in days following a Golden Cross as long as its 50DMA stays above 200 DMA;

3. At the close of the day of Black Cross (50DMA < 200DMA), the long *SPTR* position is sold completely and cash is all invested in 3-month Treasury bills;

4. 3-month Treasury bills are held as long as 50DMA stays below 200DMA for *SPTR* Index.

It is obvious that *SPTR GC-LEO* has a beta of 1.0 with full market exposure in the bullish period following a Golden Cross, and a beta of zero with no market exposure in the bearish period following a Black Cross. Table 1 shows that during the period of *21.6* years (6/1/1988 to 12/31/2009), Golden Cross and Black Cross on *SPTR* happened alternately for a total of 19 times. The longest bullish period (from Golden Cross to Black Cross triggers) lasted 1032 trading days (8/31/1994 to 10/1/1998), while the shortest bullish period only lasted a month (21 trading days between 4/19/2002 and 5/20/2002). The *SPTR* period return between Golden Cross and Black Cross trigger days are also listed in Table 1. For bullish period, the Golden Cross signal is indicated as “right” if the period return following the signal (until the next Black Gross triggers) is positive, and “wrong” if otherwise; for bearish period, the Black Cross signal is indicated as “right” if the period return following the signal (until the next Golden Cross triggers) is negative, and “wrong” if otherwise.

Table 2 indicates the effectiveness of the *SPTR* Golden Cross and Black Cross signals. Golden Cross signals were right 80% of the time and a right Golden Cross bullish signal were historically ensued by over-whelming upside (37% on average) compared to a wrong Golden Cross signal’s downside move (average 4.6% loss). Black Cross signals, though right only one-third of the time (3 out of 9), historically can avoid a bear market loss (average 22%) every time it is right. Six wrong Black Cross signals miss on average a market gain each time of just 8%. The delay in responding to a recovery from a bear market seems more serious than delay in exit long position before a major decline. The problematic Black Cross signals were on 9/17/1990 and on 10/1/1998 (the worst) that during the subsequent 101 and 45 trading sessions, SPTR actually gained 14.7% and 19.5% respectively.

Interestingly, it has been noticed that all the two wrong Golden Cross and six wrong Black Cross signals have *ex post* duration less than 101 days; and the right signals all lasted over 200 days. Overall, the Golden Cross and Black Cross market timing system is effective in capturing most of the rising market and avoiding bear market declines.

For period of 21.6 years (6/1/1988-12/31/2009), Table 3 shows the performance of *SPTR GC-LEO* with an annualized return of 12.36% (better than that of *SPTR, BXM* or *PUT*) and a Sharpe ratio of 0.68, similar to what *PUT* has and better than *SPTR* and *BXM*.

A Continuous Model for Active Leveraged Option Overlay Portfolio (*ALOOP*)

Based on Golden Cross and Black Cross over an underlying equity *S*, a two-state portfolio *Π* is proposed that writes at-the-money put options in bullish period or ATM call options in a bearish period. The number of option contracts makes its at-the-money face value equal the portfolio value, which is given in an investable form of equation (1):

$$\Pi = \begin{cases} f_l S + (1-f_l)B - P, & When \quad 50DMA \geq 200DMA \quad on \quad S \\ (1-f_l)S + f_l B - C, & When \quad 50DMA < 200DMA \quad on \quad S \end{cases} \tag{1}$$

The model is posed as continuous which implies among three components of the portfolio (*S, B, and P* or *C)* the rebalance interval is small compared to the time scale needed for underlying equity *S* to move a non-trivial amount. The Factor of Leverage f_l decides the number of shares of underlying equity and the amount of risk free Treasury bills *B* held under both bullish and bearish expectations. It is interesting to point out when $f_l = 0.5$, the portfolio essentially holds stable positions in underlying equity and Treasury bills, and the only active component of the portfolio is switching between written call and put options when Golden Cross or Black Cross triggers. When $f_l = 0$, the portfolio reduces to switching between a standard Collateralized Put-Write (*CPW*) and Covered Buy-Write (*CBW*) as expressed in equation (2):

$$\Pi = \begin{cases} CPW = B - P, & When \quad 50DMA \geq 200DMA \quad on \quad S \\ CBW = S - C, & When \quad 50DMA < 200DMA \quad on \quad S \end{cases} \tag{2}$$

Further, equation (1) can be rewritten as:

$$\Pi = \begin{cases} CPW + f_l(S - B), & When \quad 50DMA \geq 200DMA \quad on \quad S \\ CBW - f_l(S - B), & When \quad 50DMA < 200DMA \quad on \quad S \end{cases} \tag{3}$$

The above equation (3) describes an *ALOOP* in a more symmetrical form: at Golden Cross / Black Cross trigger days, the swaps are between *CPW* and *CBW*, and between a net long and short positions of f_l portion of *(S - B)*, a sub-portfolio of underlying equity position funded by borrowed Treasury bills. The advantage of equation (3) is that when applied to a standard underlying equity market index (such as *SPTR*), existing Buy-Write and Put-Write indexes (such as *BXM* and *PUT*) can be used without having to use individual option prices to evaluate the portfolio daily.

The Greek Letter Δ of the *GCBC-ALOOP* can be obtained from equation (1)

$$\Delta == \frac{\partial \Pi}{\partial S} = \begin{cases} (1 + f_l) - \delta, & When \quad 50DMA \geq 200DMA \quad on \quad S \\ (1 - f_l) - \delta, & When \quad 50DMA < 200DMA \quad on \quad S \end{cases} \tag{4}$$

where δ is delta of the call option and put-call parity is assumed valid. Further, any potential American calls or puts involved are assumed close to European options with underlying equity price or strike price much higher than zero and a small dividend yield. Further, a Taylor expansion (about $\sigma_{BS}\sqrt{T}$) gives the following expression to evaluate Δ (see **Appendix** for details of the derivation, the volatility skew assumption and related results on *SPX* and *SPTR*):

$$\Delta = \frac{\partial \Pi}{\partial S} = \begin{cases} (\frac{1}{2} + f_l) + \frac{1}{\sqrt{2\pi}}\sigma_{BS}\sqrt{T}\left[-\frac{\partial \ln \sigma_{BS}}{\partial \ln S} - \left(\frac{1}{2} + \frac{r_f - q}{\sigma_{BS}^2 T} \right) \right] + o(\sigma_{BS}\sqrt{T}), & S \quad 50DMA > 200DMA \\ (\frac{1}{2} - f_l) + \frac{1}{\sqrt{2\pi}}\sigma_{BS}\sqrt{T}\left[-\frac{\partial \ln \sigma_{BS}}{\partial \ln S} - \left(\frac{1}{2} + \frac{r_f - q}{\sigma_{BS}^2 T} \right) \right] + o(\sigma_{BS}\sqrt{T}), & S \quad 50DMA < 200DMA \end{cases} \quad (5)$$

When $f_l = 0$ equation (5) indicates *CBW* and *CPW* has the same Δ. Calibrated with *SPX* monthly options (see Appendix for details), $\Delta = 0.596$ on average for 1990-2009, right between *BXM*'s beta *0.636* and *PUT's* beta *0.573* for the same period of two decades (which are slightly different from the betas for period from 6/1/1988 to 12/31/2009 in Table 3). The deviations are mostly due to the minor out-of-money-ness of *SPX* options used in *BXM* and *PUT*, and the fact that their options and other components are rebalanced monthly rather than daily or continuously.

When $f_l = 0.5$, equation (5) gives Δ of an *ALOOP* that is always larger than that of a *GC-LEO* in either bullish or bearish periods. This can be explained by the equity *volatility skew*, or an equivalent fact that implied volatility rises when underlying equity drops ($\frac{\partial \ln \sigma_{BS}}{\partial \ln S} < 0$ in equation (5) such that the linear term coefficient is positive). When Golden Cross (or other timing signals) works right as a bullish signal, the *ALOOP* can out-perform the *GC-LEO* or Buy-and-Hold because of its equity over-exposure. In contrast, Black Cross can out-perform *GC-LEO* when not working right as a bearish signal for the same over-exposure reason.

On the other hand, equity over-exposure for *ALOOP* ($f_l = 0.5$) can lead to under-performance comparing to *GC-LEO,* in periods when underlying equity is declining but timing signal stays bullish thus delayed to respond. A sharp equity market decline following a prolonged bull run is a performance pitfall for *ALOOP*. Under market stress, volatility skew can go extreme, for example, *SPX* monthly $\frac{\partial \ln \sigma_{BS}}{\partial \ln S} = -7.6$ with $\sigma_{BS} = 33\%$ to cause the effective Δ = *beta* ~ *1.29* on *8/21/1998* (see Appendix), to expose the *ALOOP* to the risk of under-performing Buy-and-Hold; or for example, *SPX* monthly $\frac{\partial \ln \sigma_{BS}}{\partial \ln S} = -14.6$ with $\sigma_{BS} = 32\%$ to cause the effective Δ = *beta* ~ *0.54* on *9/19/2008* (see Appendix) to cause an equity market over-exposure that under-performs *GC-LEO*.

In practice, *ALOOP* ($f_l = 0.5$) cannot be continuously rebalanced to roll the written option contracts forward. However, since the over-exposure of *ALOOP* (over *GC-LEO*) increases with the time to option expiration *T*, more frequent rolling over of written put options to be at-the-money (or even out-of-money) can potentially alleviate the performance pitfall during precipitous market decline. At roll-over, the losses from written options' in-money-ness are taken immediately rather than waiting until expiration, while the collected time-value of the new at-the-money option becomes richer due to increased implied volatilities.

When a bearish timing signal (Black Cross) is delayed to switch to ride a fast market rebound, the *ALOOP* ($f_l = 0.5$) will have an under-performance relative to Buy-and-Hold position in the underlying equity. A timely roll-over of written call options can alleviate the under-performance from a prolonged bearish Buy-Write bias of the *ALOOP*. As market rises and implied volatilities on the underlying equity drops, it cost less for the time value of existing written calls and we may still collect more from the premium of the new at-the-money calls [7].

Implementation & Results for S&P 500 Total Return Index (*SPTR*) *GCBC-ALOOP*

When the S&P 500 Total Return Index (*SPTR*) is chosen as the underlying equity for a *GCBC-ALOOP*, equation (3) becomes:

$$\Pi_{SPTR} = \begin{cases} PUT + f_l \cdot SPTR - f_l \cdot B, & when \quad SPTR \quad 50DMA \geq 200DMA \\ BXM - f_l \cdot SPTR + f_l \cdot B, & when \quad SPTR \quad 50DMA < 200DMA \end{cases} \tag{6}$$

For performance tracking purpose, this indicates that *SPTR GCBC-ALOOP* can be constructed from four component indexes: *SPTR, BXM, PUT* and three-month Treasury bills *B*. Since *BXM* and *PUT* indexes re-balance among *SPTR,* T-*Bills* and written options monthly on third Friday, *SPTR GCBC-ALOOP* also rebalances among three components on option expiration Friday every month. The period return of the *SPTR GCBC-ALOOP* is given by:

$$R_t = \begin{cases} \dfrac{(PUT_t - PUT_{t-1}) + f_l \cdot (SPTR_t - SPTR_{t-1}) - f_l \cdot (B_t - B_{t-1})}{\Pi_t}, & when \quad SPTR \quad 50DMA \geq 200DMA \\ \dfrac{(BXM_t - BXM_{t-1}) - f_l \cdot (SPTR_t - SPTR_{t-1}) + f_l \cdot (B_t - B_{t-1})}{\Pi_t}, & when \quad SPTR \quad 50DMA < 200DMA \end{cases} \tag{7}$$

where Π_t is portfolio value from equation (6). Π_t includes option premiums collected and fully re-invested in *SPTR* index and 3-month *T-bills* on monthly option expiration Fridays as implied in *BXM* and *PUT* indexes.

Following a Golden Cross or Black Cross until the next option expiration Friday, the number of *SPX* option contracts and *SPTR* shares held has small but subtle difference between the approaches using *BXM / PUT* as in equation (6) and using *SPX* option contracts directly as in equation (1). Using *BXM /PUT,* the composition of option contracts and *SPTR / T-Bills* shares were decided from last option expiration Friday pricing; while with direct option approach, it is decided on basis of pricing on Golden Cross or Black Cross day. For convenience of back test, the current study uses the *BXM/PUT* approach of equation (6) without the need of daily option prices.

The back-test starts from 6/1/1988, when *PUT* and *BXM* started[2] both with nominal value of 100, and ends on 12/31/2009. On Golden Cross or Black Cross days, *SPTR GCBC-ALOOP* switches between one share of *PUT* and *BXM*, between long and short f_l share of *SPTR*, and between short and long f_l portion of *B*. There are 19 Golden/Black cross signal days during the back-tested 21.6 year period. Besides the same monthly option contracts roll-over and portfolio rebalance scheme used in *BXM* or *PUT*[5], it leads to 18 additional trades as one Golden Cross day (4/19/2002) happened to be an option expiration Friday.

At $f_l = 0$, *SPTR GCBC-ALOOP* is simply switching between *BXM* and *PUT* Indexes according to Golden Cross or Black Cross on *SPTR.* Due to about 20% margin requirement on writing *SPX* options, the Factor of Leverage f_l is capped at *0.8*. At $f_l = 0.5$, *SPTR GCBC-ALOOP* from equation (1) is approximately:

$$\Pi_{SPTR} = 0.5 \cdot SPTR + 0.5 \cdot B - \begin{cases} P, & When \quad SPTR: 50DMA \geq 200DMA \\ C, & When \quad SPTR: 50DMA < 200DMA \end{cases} \qquad (7)$$

which essentially buy-and-hold half the portfolio in *SPTR* index and another half in 3-month Treasury bills. Besides the monthly rebalance and *SPX* option contracts roll-over, the only trading is swapping into written puts from written calls on Golden Cross day close or vice versa on Black Cross day close. According to zero order portfolio Delta from equation (5), *SPTR GCBC-ALOOP* ($f_l = 0.5$) has nominal beta of 1 under bullish expectation and beta of zero under bearish expectation. This makes *SPTR GC-LEO* a natural benchmark to compare with.

Performance comparison for the 21.6 years period (6/1/1988 – 12/31/2009) is presented in Table 3 for *SPTR GCBC-ALOOP* (f_l*=0.5), SPTR* (Buy and Hold), *BXM, PUT* Indexes and the active *SPTR GC-LEO*. *SPTR GCBC-ALOOP* achieves a compounded annualized return of *16.3%*, a Sharpe ratio of *0.95*, a Sortino ratio of *1.35,* an USharpe ratio

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[2] CBOE has retrofitted both *BXM* and *PUT* indexes to start from 06/30/1986 with values less than 100 [5]. However, current study chooses not to include results of *SPTR GCBC-ALOOP* from those early data for performance comparison. This is because during market crash of 1987, some basic assumptions about option pricing and derivative market efficiency was well documented as questionable.

of 48%, all significantly better than those of the passive indexes: Buy-and-Hold *SPX Total Return (SPTR)*, *PUT*, and *BXM*, and those of *SPTR GC-LEO*. The maximum drawdown *22.88%* of *SPTR GCBC-ALOOP* is slightly higher than that of the *SPTR GC-LEO (19.10%)*, but much smaller than those of the passive indexes. For the 21.6-year period, Beta of *SPTR GCBC-ALOOP* turns out to be *0.556,* which is larger than that of *SPTR GC-LEO* (*0.458),* but smaller than those of *BXM* and *PUT.* Annualized standard deviation for *SPTR GCBC-ALOOP* (*14%*) is larger than those of *BXM, PUT* and *SPTR GC-LEO*, but quite smaller than that of *SPTR*. Reflected in Sortino ratio comparison, larger standard deviation of *SPTR GCBC-ALOOP* is the result of larger upside return bias rather than downside risk.

Dollar growth comparison is presented in Figure 1a (linear scale) and 1b (log-scale). *SPTR GCBC-ALOOP* can grow from one dollar in 6/1/1988 into *26.22* dollars on 12/31/2009, compared to *10.36* dollars for *SPTR GC-LEO*, *9.73* dollars for *PUT*, *7.66* dollars for *BXM*, and only *6.77* dollars for Buy-and-Hold *SPTR.*

SPTR GCBC-ALOOP year by year returns are presented in Table 4 and Figure 2 in comparison with *SPTR, BXM, PUT* and *SPTR GC-LEO*. Thirteen out of twenty-two years (06/01/1988-12/31/1988 as a partial year), *SPTR GCBC-ALOOP* has the best annual return among the five compared passive indexes and active strategies.

Buy-and-Hold *SPTR* was the best performer in four years: 1991, 1995, 1998 and 2003; *SPTR GC-LEO* was the best performer in three years: 1995 (tied with *SPTR*), 2001 and 2008; *PUT* was the best performer in three years: 1990, 1994 and 2000. Double digit under-performance only happened three years for *SPTR GCBC-ALOOP*:

1. During 1990, Golden Cross and Black Cross timing signals misfired three times in a row (see Table 1). *SPTR GCBC-ALOOP* still managed to out-perform *SPTR* Buy-and-Hold and *SPTR GC-LEO* by *1%* and *6%*, respectively in 1990 but under-performed *PUT* by *11%* during a volatile but flat year.
2. Spiking volatilities in 1998 (*SPX* about 20% drop in 31 trading days from 7/17/1998 to 8/31/1998, and followed by over 25% bull run till year end) turned out to be challenging for Golden Cross/Black Cross timing scheme. Black Cross triggered on 10/1/1998 and bearish bias lasted until 12/4/1998 while *SPTR* rose almost 20% in the period. *SPTR GCBC-ALOOP* out-performed *SPTR GC-LEO* by 4.1% in 1998. Volatility skew led to *SPTR GCBC-ALOOP* having about 18% over-exposure during 7/17/1998 to 8/31/1998, and it under-performed *SPTR* Buy-and-Hold by 3.8% for the period. Weekly rather than monthly written options roll-over could

have alleviated the situation. *SPTR GCBC-ALOOP* under-performed *SPTR* Buy-and-Hold by *16.4%* in 1998.

3. Despite out-performing *SPTR* Buy-and-Hold by *27.6%* in 2008, *SPTR GCBC-ALOOP* under-performed *SPTR GC-LEO*'s T-bills position in 2008 by *10.4%*. Volatility skew caused over-exposure to market. To the extreme on 9/19/2008, *SPTR GCBC-ALOOP* has an effective beta of *0.54* while the benchmark *SPTR GC-LEO* dictates zero market exposure throughout 2008.

__Further Discussion on SPTR GCBC-ALOOP__:

So far all the *SPTR GCBC-ALOOP* results presented have been based on "Factor of Leverage" $f_l = 0.5$. Treating f_l as a parameter, it is found that at $f_l \approx 0.575$, *SPTR GCBC-ALOOP* has the optimal risk-adjusted-return (see Figure 3 and Table 5) for the 21.6 years period. $f_l = 0.8$ is the limit of leverage for *SPTR GCBC-ALOOP* due to margin requirements on *SPX* options. $f_l = 0$ means simple swap between *BXM* and *PUT* according to Golden Cross or Black cross on *SPTR.* Notice the flatness of curve for risk-adjusted-return measures (Figure 3) when $f_l \geq 0.5$. *SPTR GCBC-ALOOP* ($f_l = 0.5$) has nearly best risk-adjusted-returns and involves no nominal trading in *SPX* or T-bills (from equation (7)) when active timing signals are triggered. Active trading for *SPTR GCBC-ALOOP* ($f_l = 0.5$) is limited to once a year on average that involves swapping written *SPX* calls all into written *SPX* puts, or vice versa, as such it is chosen as the baseline case for *SPTR GCBC-ALOOP* analyses. $f_l = 0.8$ maximizes portfolio leverage and achieves 3% per annum extra return compared to $f_l = 0.5$.

The monthly roll-over of written options inherited from the methodology of *BXM* and *PUT* also makes market friction and transaction cost a potential concern for *SPTR GCBC-ALOOP* ($f_l = 0.5$). From 6/1/1988 to 12/31/2009, *259* times portfolio rebalancing and option roll-over are needed at monthly option expiration Fridays and *18* market timing trades of swapping written puts and calls on Golden Cross/Black Cross days. Assume *2* basis points of spread is involved in each option transaction (two transactions per roll-over or market timing trades) and *1* basis point can cover transaction fee and rebalancing trades (reasonable for portfolio size over one million dollars). Total *5* basis points (denoted as $F = 5bp$) is a relatively conservative estimate of total market friction (before tax or any management fee) for every one of the *277* transactions for *SPTR GCBC-ALOOP* ($f_l = 0.5$).

As indicated in Table 6 and Figure 4, the out-performance of *SPTR GCBC-ALOOP* ($f_l = 0.5$) over *SPTR GC-LEO or Buy-and-Hold* remains intact even when the transaction cost ratio is quadrupled to *F=20bp,* a situation roughly equal to *F=5bp* plus a *2.00%* per year management fee.

Final Discussion and Conclusion

The intended out-performance of *ALOOP* (over Buy-and-Hold) is based on the combination of an active management scheme, such as Golden Cross and Black Cross, and an option overlay mechanism such as Buy-Write and Put-Write. Although traditional financial economics theory and market efficiency hypothesis gives little value to both methodologies, behavioral finance [8] can offer some explanations why they could work consistently in practice. Momentum investing system (such as Golden Cross/Black Cross) indicates investor's *aversion to ambiguity*: the market participants have consensus that trending (up or down) exists in market with over 50% odds that it is better off go trend-following as momentum becomes a self-fulfilling prophecy. Option overlay methodology is built on the premise that implied volatilities are larger than realized volatilities in the underlying equity (or index). This reflects the amplified level of *loss aversion* and *over-confidence* in the option market compared to the underlying asset market, probably due to more hedging (portfolio insurance) and speculation purposed participants in the option market.

The interaction of the active management scheme and option overlay mechanism also help *ALOOP* to out-perform Buy-and-Hold approach despite pitfalls under extreme market conditions. The following table summarizes generalizations based on findings in *SPTR GCBC-ALOOP* ($f_l = 0.5$)'s 21.6 years period back test:

Active Signal Signal Outcome	Bullish Expectation to Long Bias (e.g. Golden Cross)	Bearish Expectation to be Market Neutral (e.g. Black Cross)
Right	*Out-perform both* Buy-and-Hold and Long-only Beta matched Active Strategy due to equity over-exposure from the volatility skew of written puts	*Out-perform* Buy-and-Hold due to right signal, but may *under-perform* the Long-only Active Strategy due to equity over-exposure from written calls
Wrong	May *under-perform both* Buy-and-Hold (due to wrong signal) and the Long-only Active Strategy (due to equity over-exposure)	May *under-perform* Buy-and-Hold due to wrong signal, but can *out-perform* the Long-only Active Strategy due to equity over-exposure

Due to the interaction among mechanisms, a robust active management scheme that can avoid bear market and capture most of bull-run is the key to success for *ALOOP.* For *SPTR* as the underlying equity index, it appears that Golden Cross / Black Cross has been a robust active management scheme for different levels of leverage (f_l between *0.5* and *0.8*) in the portfolio. Out-performance of *SPTR GCBC-ALOOP* (over Buy-and-Hold) is significant on both absolute return and risk-adjusted basis after fully counting market friction and transaction costs. As topic of future studies, other alternative active schemes, for example, Golden Cross and Black Cross on *VIX* could also be working well for *SPTR-ALOOP*.

Over-exposure to equity due to option volatility skew appeared to enhance out-performance of *ALOOP* most of the time. Equity volatility skew is also explained by investor behavioral trait of "*crashophobia*" (Mark Rubinstein) that was only observed after the market crash of 1987. *ALOOP* is taking advantage of the volatility skew phenomena along with its market timing scheme. The "Factor of Leverage" can also be separately defined for the bullish and bearish cases, especially allowing *ALOOP* to have net short bias in a bearish regime.

It is of interest to try to apply the *ALOOP* to other equity market indexes and even single stock position. *SPX* is highly liquid tradable index with good price smoothness (no jump) in the past 20+ years. Other index and single stock name may not have these characteristics that *ALOOP* relies on for consistent performance in practice. Actually, retrofitted back-test covering period 6/30/1986 to 6/1/1988 shows *SPTR GCBC-ALOOP* ($f_l = 0.5$) had a single day loss of *34.6%* on 10/19/1987 and *46%* draw-down at that point. Thus *ALOOP* needs an efficient functioning derivative and underlying asset market to work. All the current Greek Letter Delta related analytical work is based on the assumption that volatility is not constant but price is continuous with no jump (Jump diffusion is another situation beyond standard Black-Scholes Option Pricing Theory). However, *ALOOP* can be applied to other broad market indexes such as Dow Jones Industrial Average, Nasdaq 100 and Russell 2000 upon which CBOE has created Buy-Write Indexes, as suggested line of future work.

Given the relative easiness to implement the proposed *ALOOP* approach for index investment, it has the potential for broad adoption. Thus it is of interest to examine the future sustainability of the out-performance observed in the *21.6* years of back-test for *SPTR*. Since the active strategy proposes a fixed portion (about half) of the portfolio to Buy-and-Hold the *SPTR* index, it has a stabilization effect in the underlying asset market and potentially can slow down the pace of a market decline. This provides a natural remedy for the discussed performance pitfall of the *ALOOP* strategy itself, and help to improve the order-nesses and efficiencies in both the underlying asset and option markets. On the other hand, amplified in option market relative to asset market, investor behavioral biases of fear or over-confidence could be the fundamental reason for the active strategy's hypothetical historical success. As long as these market structural characteristics and human nature in investing persist, the observed effectiveness of the current proposal could be expected to last, even if we have not found an active management's "holy grail", for decades to come.

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[3]Whaley, R, 2002, "*Return and Risk of CBOE Buy Write Monthly Index*" Journal of Derivatives, vol. 10, no. 2, pp. 35–42.

[4] Ennis Knupp Associates, Inc., "*Evaluating the Performance Characteristics of CBOE Put-Write Index*", Ennis Knupp Associates, Inc. December 2008

[5]CBOE Index Websites, http://www.cboe.com/micro/IndexSites.aspx

[6]John J. Murphy, *Technical Analysis of the Futures Markets*, New York Institute of Finance, 1997.

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Table 1: Golden Cross/Black Cross on S&P 500 Total Return Index (*SPTR*) 1988-2009[3]

Trade Date	Type of Signal	Duration Till Next Signal (Trading Days)	SPX P/L Till Next Signal	Bullish/Bearish Right or Wrong
6/16/1988	Golden Cross (Bullish)	439	32.37%	Right
3/13/1990	Black Cross (Bearish)	41	2.76%	Wrong
5/10/1990	Golden Cross (Bullish)	89	-6.32%	Wrong
9/17/1990	Black Cross (Bearish)	101	14.71%	Wrong
2/8/1991	Golden Cross (Bullish)	815	38.68%	Right
5/2/1994	Black Cross (Bearish)	85	6.09%	Wrong
8/31/1994	Golden Cross (Bullish)	1032	126.14%	Right
10/1/1998	Black Cross (Bearish)	45	19.57%	Wrong
12/4/1998	Golden Cross (Bullish)	484	24.07%	Right
11/3/2000	Black Cross (Bearish)	361	-19.63%	Right
4/19/2002	Golden Cross (Bullish)	21	-2.87%	Wrong
5/20/2002	Black Cross (Bearish)	246	-11.87%	Right
5/12/2003	Golden Cross (Bullish)	325	19.46%	Right
8/25/2004	Black Cross (Bearish)	44	2.17%	Wrong
10/27/2004	Golden Cross (Bullish)	439	16.39%	Right
7/26/2006	Black Cross (Bearish)	24	2.94%	Wrong
8/29/2006	Golden Cross (Bullish)	335	16.29%	Right
12/28/2007	Black Cross (Bearish)	370	-35.50%	Right
6/18/2009	Golden Cross (Bullish)	136	22.91%	Right

Table 2a: Effectiveness of *SPTR* Golden Cross and Black Cross Signals

	Bullish Regime Golden Cross (Times)	Bullish Regime Golden Cross (Sum of SPX P/L)	Bearish Regime Black Cross (Times)	Bearish Regime Black Cross (Sum of SPX P/L)
Right	**8**	**296.32%**	**3**	**-67.00%**
Wrong	**2**	**-9.19%**	**6**	**48.25%**

Table 2 b: Effectiveness of *SPTR* Golden Cross and Black Cross Signals

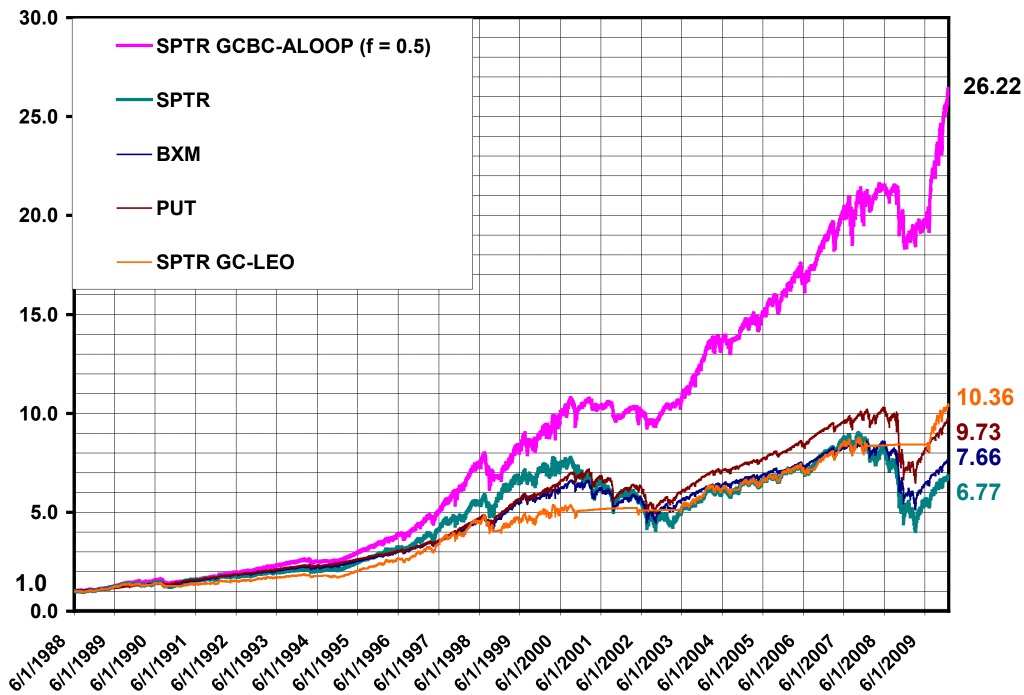
	Bullish Regime Golden Cross SPX Average P/L	Bearish Regime Black Cross SPX Average P/L
Right	**37.04%**	**-22.33%**
Wrong	**-4.60%**	**8.04%**

[3] The duration of the last Golden Cross (Bullish) signal on 6/18/2009 is calculated through 12/31/2009. The bullish expectation has continued till right before the submission of this paper 3/12/2010. The duration is already 184 trading days with a period SPX return about 26%.

Table 3: Performance Comparison of *SPTR GCBC-ALOOP* to *GC-LEO* & Passive Indexes[4]

	SPTR	BXM	PUT	SPTR GC-LEO	SPTR GCBC-ALOOP (f = 0.5)
Annualized Return	9.26%	9.89%	11.11%	12.36%	16.33%
Annualized Std Deviation	18.27%	12.74%	11.88%	11.43%	13.99%
Sharpe Ratio	0.3440	0.5425	0.6847	0.6841	0.9543
Sortino Ratio	0.4922	0.7509	0.9480	0.9899	1.3483
Beta	1.0000	0.6345	0.5693	0.4581	0.5563
USharpe Ratio	6.29%	15.57%	21.61%	25.82%	47.99%
Max Draw-Down	55.24%	40.14%	37.09%	19.10%	22.88%

Figure 1a: Dollar Growth of *SPTR GCBC-ALOOP* and Comparison to *SPTR, BXM, PUT* and *SPTR GC-LEO* (Linear Scale)



[4] Daily return data from 6/1/1988 to 12/31/2009 are used for annualized standard deviation calculation. A risk free rate $r_f = 2.97\%$ (annualized yield on 3-month Treasury bill for the same period) is used in *Sharpe* ratio, *Sortino* ratio and *USharpe* ratio calculation. A *USharpe* ratio based on annualized return r is defined as: $(r - r_f)/Ulcer$, where *Ulcer* is the ratio between the daily arithmetic average drawdown of the portfolio strategy and that of the *S&P 500 Total Return Index (SPTR)*. *USharpe* ratio is similar to UPI (Ulcer Performance Index) or Martin ratio but refers more specifically here to *SPTR*.

Figure 1b: Dollar Growth of *SPTR GCBC-ALOOP* and Comparison (Log-scale)

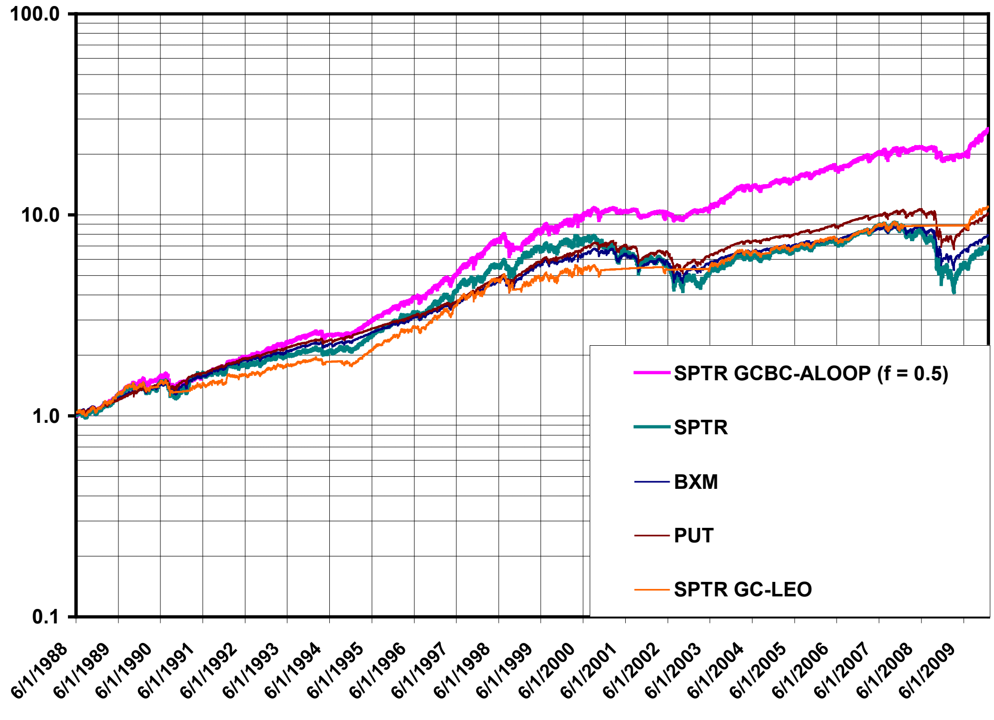


Table 4: *SPTR GCBC-ALOOP* ($f_j=0.5$) year by year returns and Comparison

	SPTR	BXM	PUT	SPTR GC-LEO	SPTR GCBC-ALOOP (f = 0.5)
1988*	6.34%	8.13%	6.90%	5.33%	9.19%
1989	31.63%	25.01%	24.58%	31.63%	38.73%
1990	-3.11%	3.99%	8.88%	-8.21%	-2.13%
1991	30.40%	24.39%	21.32%	20.20%	24.58%
1992	7.61%	11.52%	13.80%	7.61%	16.56%
1993	10.06%	14.10%	14.14%	10.06%	18.45%
1994	1.31%	4.50%	7.10%	-3.51%	1.51%
1995	37.53%	20.97%	16.88%	37.53%	34.33%
1996	22.94%	15.50%	16.40%	22.94%	26.69%
1997	33.35%	26.64%	27.68%	33.35%	44.67%
1998	28.58%	18.95%	18.54%	8.11%	12.23%
1999	20.91%	21.40%	20.98%	20.91%	30.54%
2000	-9.10%	7.40%	13.06%	-1.35%	11.84%
2001	-11.87%	-10.92%	-10.63%	2.42%	-4.06%
2002	-22.10%	-7.64%	-8.58%	-1.86%	2.67%
2003	28.67%	19.37%	21.77%	19.43%	27.35%
2004	10.87%	8.30%	9.48%	8.74%	12.62%
2005	4.91%	4.25%	6.71%	4.91%	7.94%
2006	15.78%	13.33%	15.16%	12.86%	18.48%
2007	5.49%	6.59%	9.51%	6.04%	11.14%
2008	-36.99%	-28.65%	-26.77%	0.98%	-9.39%
2009	26.47%	25.91%	31.51%	22.99%	37.86%

Figure 2: *SPTR GCBC-ALOOP* Year by year Returns and Comparison

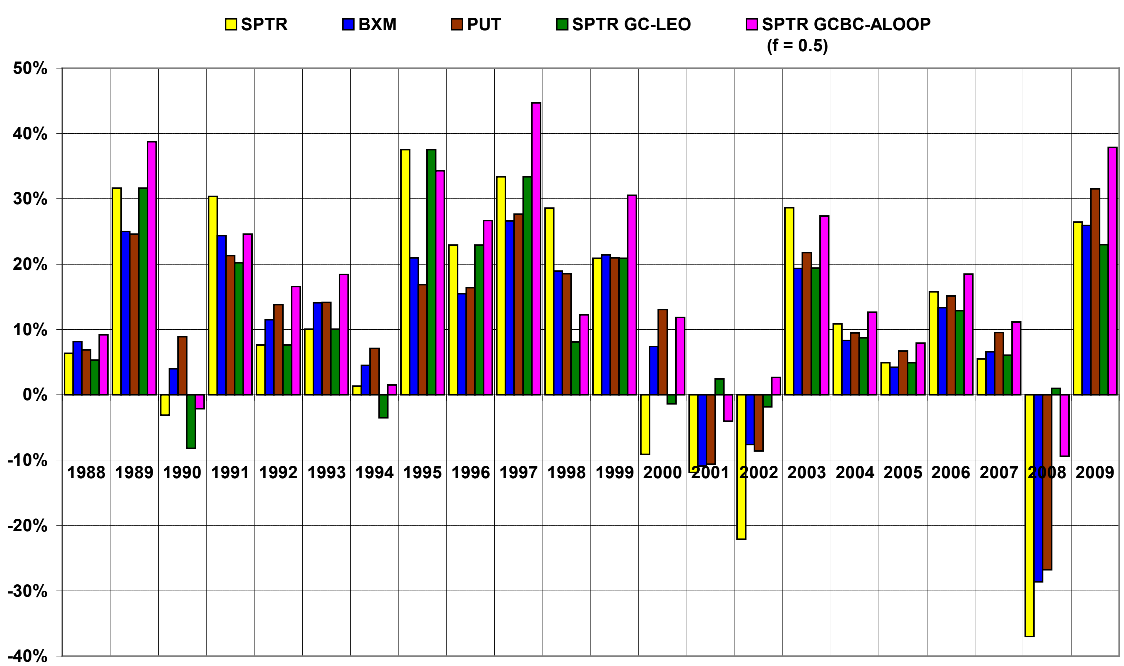


Figure 3: *SPTR GCBC-ALOOP* Risk-Adjusted-Return vs. Factor of Leverage

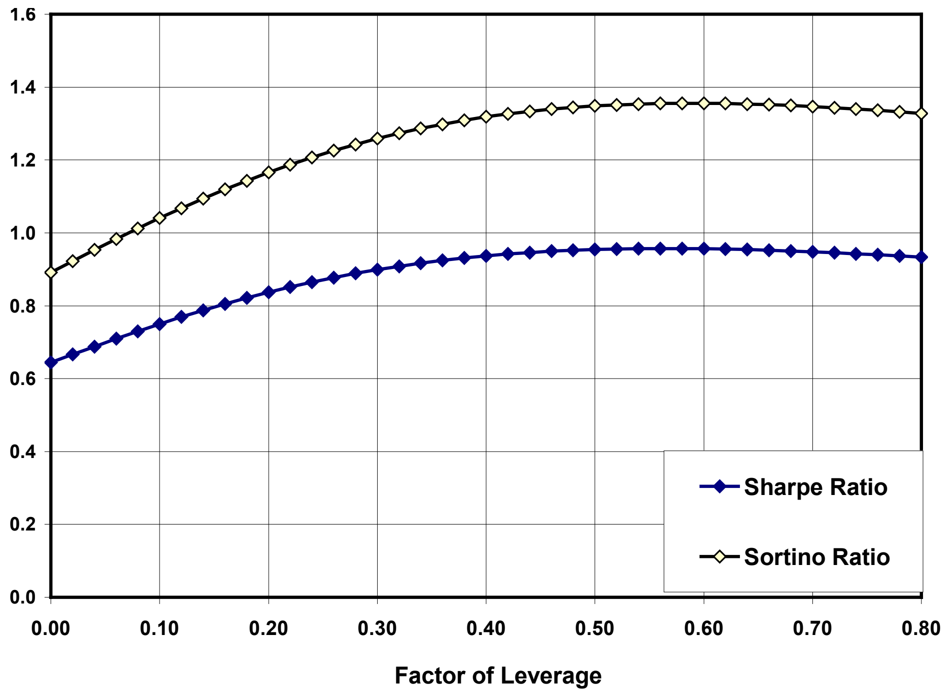


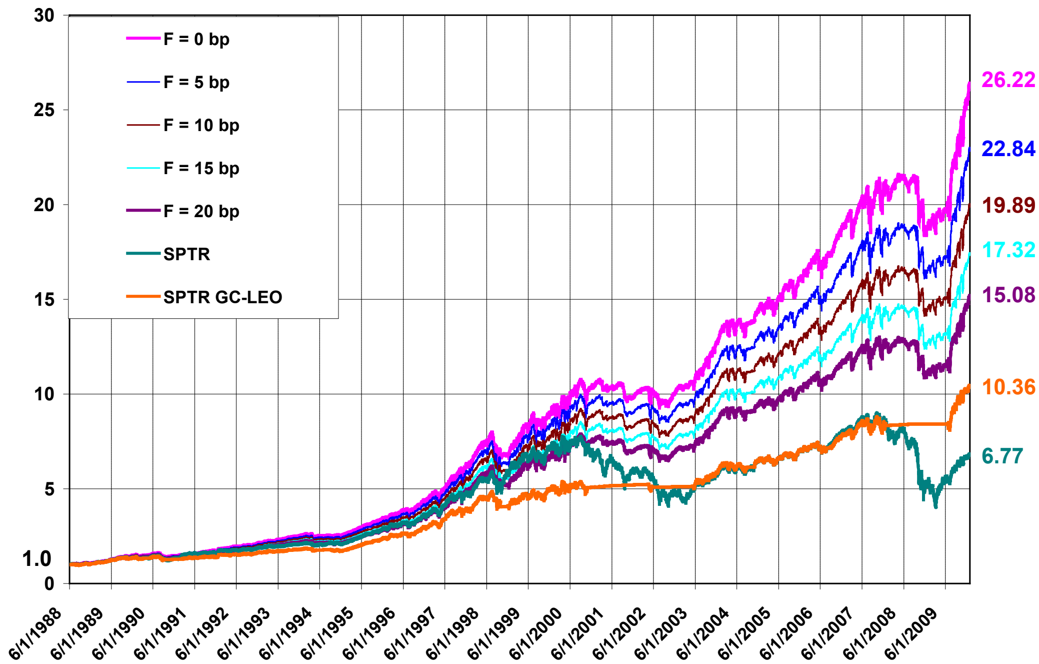
Table 5: *SPTR GCBC-ALOOP* Performance with Factor of Leverage Variations

Performance / Leverage Factor	f = 0	f =0.5	f = 0.575	f = 0.8
Annualized Return	10.86%	16.33%	17.10%	19.35%
Annualized Std Deviation	12.24%	13.99%	14.76%	17.54%
Sharpe Ratio	0.6440	0.9543	0.9570	0.9334
Sortino Ratio	0.8912	1.3483	1.3555	1.3275
Beta	0.5991	0.5563	0.5509	0.5361
USharpe Ratio	19.60%	47.99%	50.64%	50.23%
Max Draw-Down	40.21%	22.88%	24.23%	28.21%

Table 6: Impact of Transaction Costs on *SPTR GCBC-ALOOP* ($f_j = 0.5$) Performance

Performance / Cost Per Trade	0 bp	5 bp	10 bp	15 bp	20 bp
Annualized Return	16.33%	15.58%	14.85%	14.12%	13.39%
Annualized Std Deviation	13.99%	14.00%	14.01%	14.02%	14.04%
Sharpe Ratio	0.9543	0.9008	0.8476	0.7946	0.7419
Sortino Ratio	1.3483	1.2707	1.1936	1.1172	1.0413
Beta	0.5563	0.5566	0.5569	0.5571	0.5574
USharpe Ratio	47.99%	42.56%	37.72%	33.32%	29.23%
Max Draw-Down	22.88%	22.96%	23.04%	23.13%	23.21%

Figure 4: Dollar Growth of *SPTR GCBC-ALOOP*: Transaction Cost Variations



Appendix: Estimation of Greek Letter Delta under Volatility Skew

Under Black-Scholes Option Pricing Theory, the Delta of a European call option [7] is: $\delta_{BS} = \exp[-q \cdot T] \cdot N(d_1)$ where $N(d_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d_1} \exp(-x^2/2) dx$ and

$d_1 = [\ln(S/K) + (r_f - q)] \big/ (\sigma_{BS}\sqrt{T}) + \sigma_{BS}\sqrt{T}/2$

When also consider existence of volatility skew:

$\delta = \delta_{BS} + \frac{\partial C}{\partial \sigma_{BS}} \cdot \frac{\partial \sigma_{BS}}{\partial S}$

And $\frac{\partial C}{\partial \sigma_{BS}} = \nu = \frac{1}{\sqrt{2\pi}} S\sqrt{T} \exp[-d_1^2/2 - q \cdot T]$

Consider the case of at the money call *K=S* and $d_1 = [1 + 2 \cdot (r_f - q)/(\sigma_{BS}^2 \cdot T)] \cdot \sigma_{BS}\sqrt{T}/2$

when $(r_f - q)/(\sigma_{BS}^2 \cdot T) \sim O(1)$ and $\exp[-q \cdot T] \approx 1$, a Taylor expansion about

$\sigma_{BS}\sqrt{T} \sim o(1)$ for δ :

$$\delta = \tfrac{1}{2} - \tfrac{1}{\sqrt{2\pi}} \sigma_{BS}\sqrt{T}[-\frac{\partial \ln \sigma_{BS}}{\partial \ln S} - \tfrac{1}{2} - (r_f - q)/(\sigma_{BS}^2 \cdot T)] + o(\sigma_{BS}\sqrt{T}) \qquad (8)$$

For SPX monthly ($T = \frac{1}{12}$) call options for the period of VIX started to be available on 1/2/1990 to 12/31/2009, on average, *VIX*= $\sigma_{BS} \sim 20\%$, $r_f = 2.8\%$, and $q = 3.4\%$ (in annualized terms) satisfy the expansion conditions. Thus $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$, as a representation of volatility skew (how much *VIX* change relative to change in SPX index) becomes critical to determine first order effect (how different from one half) for at the money call options.

Excluding the months that month-to-month SPX change less than 1% (a singularity situation for $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$), the distribution of $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ is shown in Figure A1 and A2. In the 187 months sampled, it has a mean of *-3.58*. Thus on average it is estimated that $\delta = 0.404$ for 20 year period (1990-2009).

From equation (8) for *SPX*, it is estimated that if $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S} < -2.5$, the first order effect in Delta is comparable to zero order level (*1/2*). Historical monthly $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ are plotted in Figure A3 where instances of below -2.5 are indicated below the red line. If the active timing signal is right at the time, they will cause *SPTR GCBC-ALOOP* ($f_l = 0.5$) to have favorable over-exposure to the equity market. Otherwise, these cases will cause unfavorable over-exposure. They lead to under-performance of *SPTR GCBC-ALOOP* ($f_l = 0.5$) that the recent case has been on 9/19/2008, and previous one on 8/21/1998.

Figure A1: $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ **(Monthly Change in *VIX* divided by Change in *SPX*) in 1990-2009**

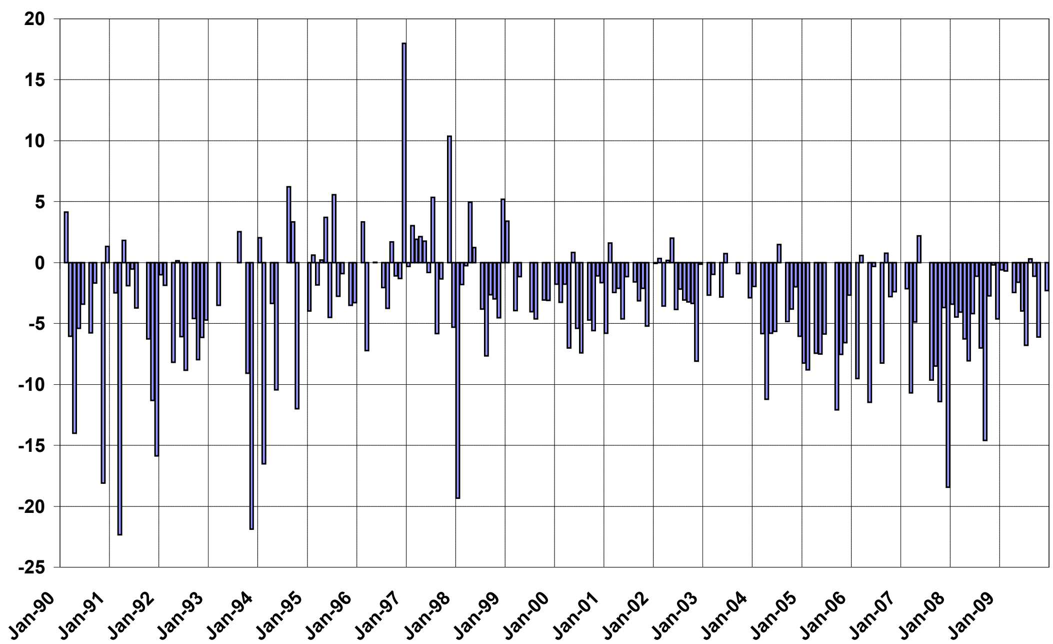


Figure A2: Histogram of $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ in 1990-2009 (Mean = *-3.58*)

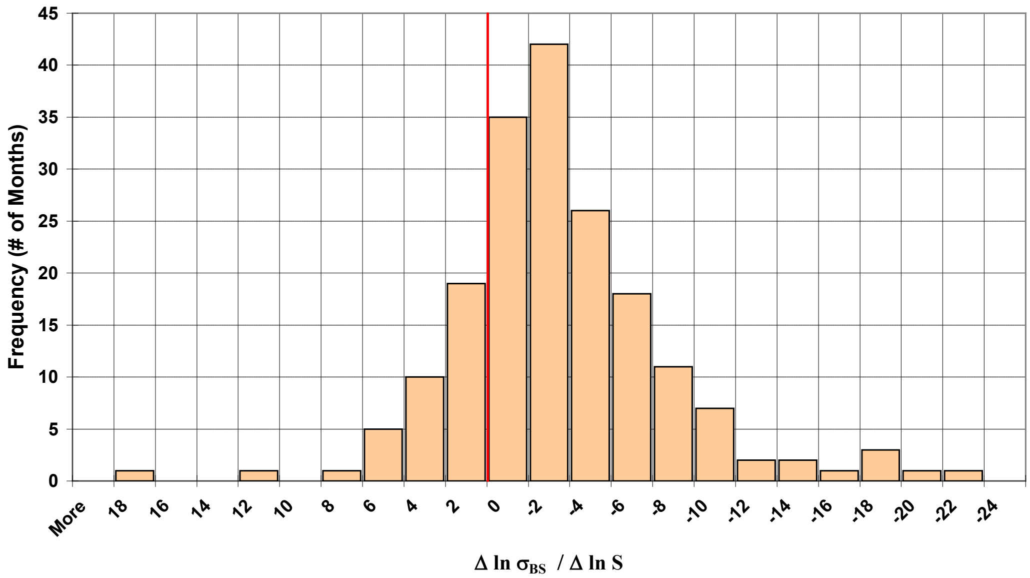


Figure A3: **Monthly $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S} < -2.5$ cases in 1990-2009:**

Periods of Potential Unfavorable *SPTR GCBC-ALOOP* Over-exposure